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Symmetry in Armenian Mediaeval Ornaments

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Abstract

The Armenian mediaeval culture is a treasure of ornaments. Sacral architecture, stone and wood carvings, decorations on paper and fabric, all contain planar periodic patterns. For the first time we classify the available ornaments according to their symmetry properties with the tools of mathematical group theory. We determine the unit cells of patterns, axes of rotation, mirror and glide reflections, all symmetry operations that preserve the pattern invariant. The results show that of seventeen crystallographic plane groups, four do not exist. Other symmetry groups are represented in a more or less balanced distribution with a dominance of fourfold symmetry. The distribution of the symmetry groups can be used to rigorously compare different cultural groups. Armenia, with its geographical location along the Silk Road, was inspired by different cultures and served as a source of inspiration for many cultures. The mathematical analysis of ornaments is an objective measure to follow such interactions.

Keywords

Armenian Mediaeval Art, Ornaments, Symmetry, Wallpaper Group, Crystallographic Plane Groups, Periodic Patterns

Introduction

Decorations and ornaments, especially regularly tessellated patterns, are a collection of intelligent signs created by men from a cultural group. They are means of expression and basic tools of human skills that are passed on to others. This information is characteristic of the group of craftsmen or architects who express their state of mind, which is developed and accumulated over time. Therefore, the ornaments can be used to identify an ethnic or cultural group by examining the symmetries they exhibit. To this end, an attempt was made to identify the Byzantine ornamentation in St. Marc's Basilica in Venice (Erbudak 2019).

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The classical school uses the decorative elements, the harmony of colours and shapes as key criteria for the classification of ornaments. This approach has the advantage of preserving the beautiful details of the artwork. Owen Jones (2001) classified the ornaments in his encyclopaedic work by the ethnic or cultural groups in which they were created. According to him, "Ornament is… the very soul of an architectural monument".

In this report, we will try to show that Jones' statement is particularly true of Armenian architecture, where a building cannot be designed without appropriate ornamentation or even everyday objects. We are aware that Armenian architectural work is most often associated with stonemasonry (Kyurkchyan/Khatcherian 2010). The affinity to stones can come from the mountainous location of the country: stone is the most common building material in Armenia. But Armenian art is not only limited to stone carving, but also to wood and metal carvings, ceramic designs, textile, and paper ornaments (Kyurkchyan 2016). We present examples of these works of art, including their graphical abstraction. The mathematical analysis of Armenian artifacts is the original contribution of this report, which allows an objective comparison of Armenian culture with others.

SACRAL ARCHITECTURE

As a striking example to Armenian architecture, we visit Surb Khach Church, an Armenian mediaeval building on Akhtamar Island on Lake Van in eastern Turkey. It is an exceptional monument. The exterior walls of the 10th century building are decorated with human, animal, and plant figures, as well as linear and planar ornaments in the form of stone bas-reliefs. The decorations and reliefs, which complement the architectural building are illustrated (İpşiroğlu 1963), as well as the iconographic myths and legends, which are conveyed in detail (Davies/Kersting 1991; for a general overview and important examples of Armenian ornamentation, see Kyurkchyan/Khatcherian 2010). Here, it is our goal to identify only the regular ornaments as part of the decoration.

Figure 1 shows the eastern part of the south façade of the building. The first available report is from the beginning of the 20th century and is an excellent documentation of the time when the Armenian artwork in eastern Turkey was still partly intact (Bachmann 1913). On this façade we observe on the left two rabbits and two bears, all rearing. A fabulous lion with wings is above the animals. We discern an eightfold *rosette* on each of his legs. Above this griffin, framed in a sixtyfold rosette and a halo of similar type, we see a religious figure. Further to the right, under the window, there is a saint with a halo similar to the previous one and a long cape. The fabric shows a planar, regular (two-dimensional) pattern. The second saint on the right also wears a long cloak with a fourfold-symmetrical ornament. His halo has the characteristic shape of a rosette.

Figure 1. The southeastern façade of the Surb Khach Church on Akhtamar Island; the entire building is covered with horizontal decorative bands at various heights, all carved in stone.

Figure 2 depicts a front view of this saint. He carries a cross in his left hand. The inscription states that he is *Saint Hamazasp, Duke of Vaspurakan*. Vaspurakan is the ancient Kingdom of Armenia during which the Surb Khach Church was built. Sahak the Parthian, on the left, is pointing with both hands to the saint; also his cape is patterned. On the right side of the figure, the planar abstraction of the ornament is shown to demonstrate the fourfold symmetry. These two saints, depicted on the outer walls of the Surb Khach Church, are frequently found in literature on Armenian culture (Bachmann 1913; İpşiroğlu 1963; Davies/Kersting 1991).

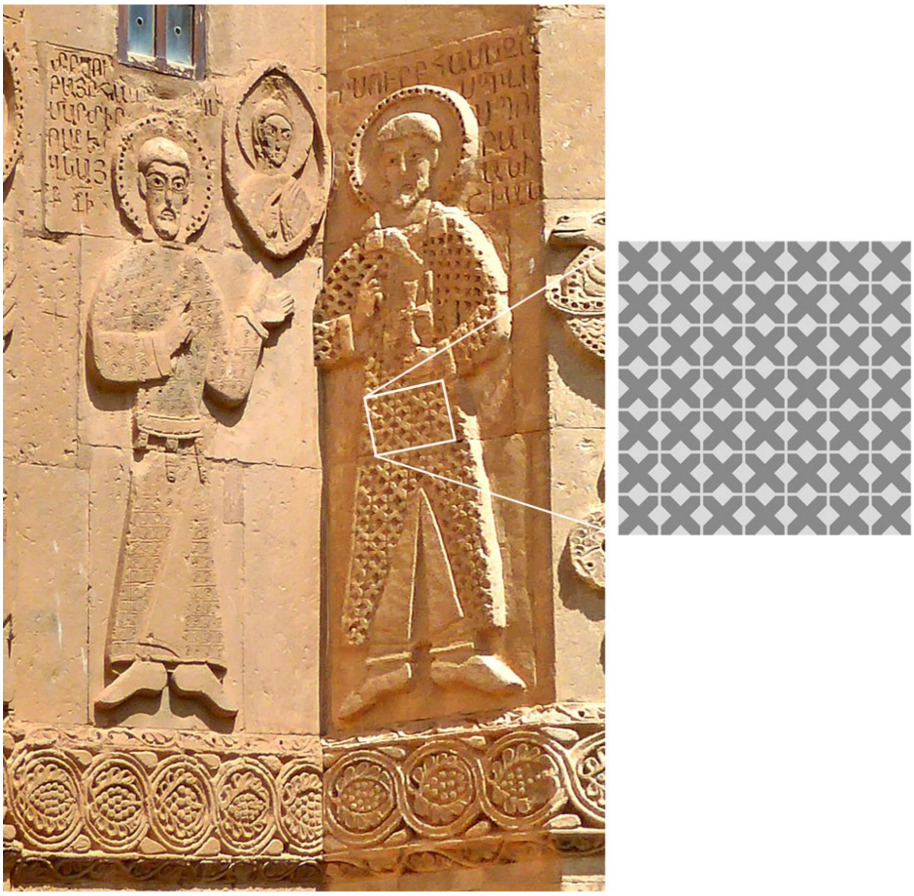

Figure 2. A close-up view of the Saint Hamazasp, Duke of Vaspurakan, with decorated cape, whose fourfold symmetry can be seen on the right side. For the symmetry analysis of the highlighted pattern (*p4mm*), see the text. Sahak the Parthian, the Armenian Catholicos, is on the left.

On Figure 1, next comes a bird with a ram head. Also, its wings and chest feathers have a regular pattern. Below this fantastic animal we see an eagle with its prey. Its feathers are patterned too. Above the animal is the portrait of another saint. There is a beautifully curved frieze above the window on the right. The bird above the window has patterned feathers. The next standing figure wears a coat with interesting regular patterns. Further to the right, between David (holding a sling in his right hand) and Goliath (equipped with a sword and a shield) and above the sleeping deer, there are two more rosettes with sixfold and twelvefold symmetry. The giant Goliath wears armour consisting of several parts, each with an ornament of different symmetry. He carries a shield with two concentric rosettes, the larger with seventy-two and the smaller one with twelvefold symmetry. Saul (first king of Israel) to David's left also wears a cape with regular patterns.

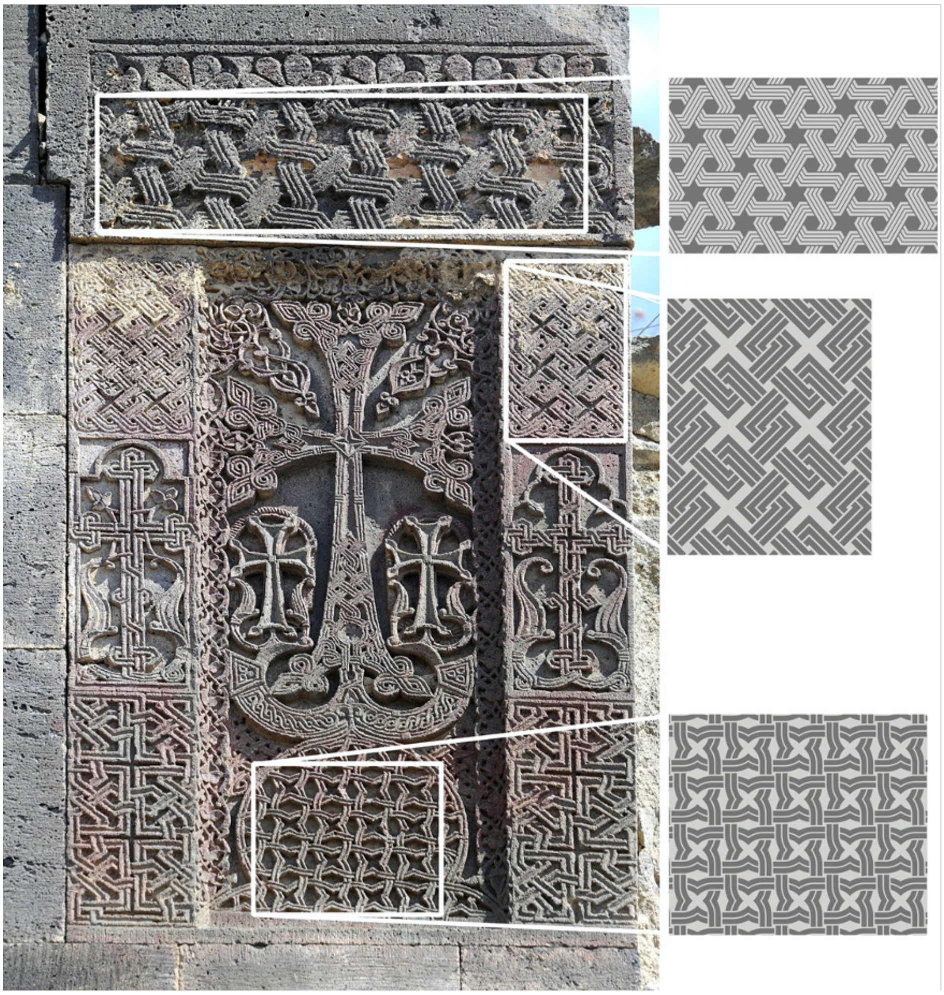

Figure 3. A tombstone in the garden of Aghjots Vank. Smaller crosses supplement the main cross in the middle. The surface is full of artistic friezes and beautiful two-dimensional ornaments. The abstractions (for the groups *p6*, *p4*, from top to bottom) are presented on the right as separate graphs.

On the façade there are two different bands of frieze. The higher one near the roof in the east is called the vine-harvest frieze (Bachmann 1913; İpşiroğlu 1963; Davies/Kersting 1991). It mainly shows a vine stock full of grapes. Inside the plant we see animals like a fox, a dog, hares, some birds, a sphinx, two bears among several people. At the lower edge of Figure 1, below the feet of the sculptures of animals and human figures, there is a continuous frieze of grapes individually enclosed by acanthus leaves. Above this band another one runs parallel to it, which is hardly recognisable. It is a continuous frieze of small leaves, which is better discernible in Figure 2 below the feet of the saint. These details of the façade of Surb Khach Monastery show that the Armenian sacral architecture has a wealth of ornaments.

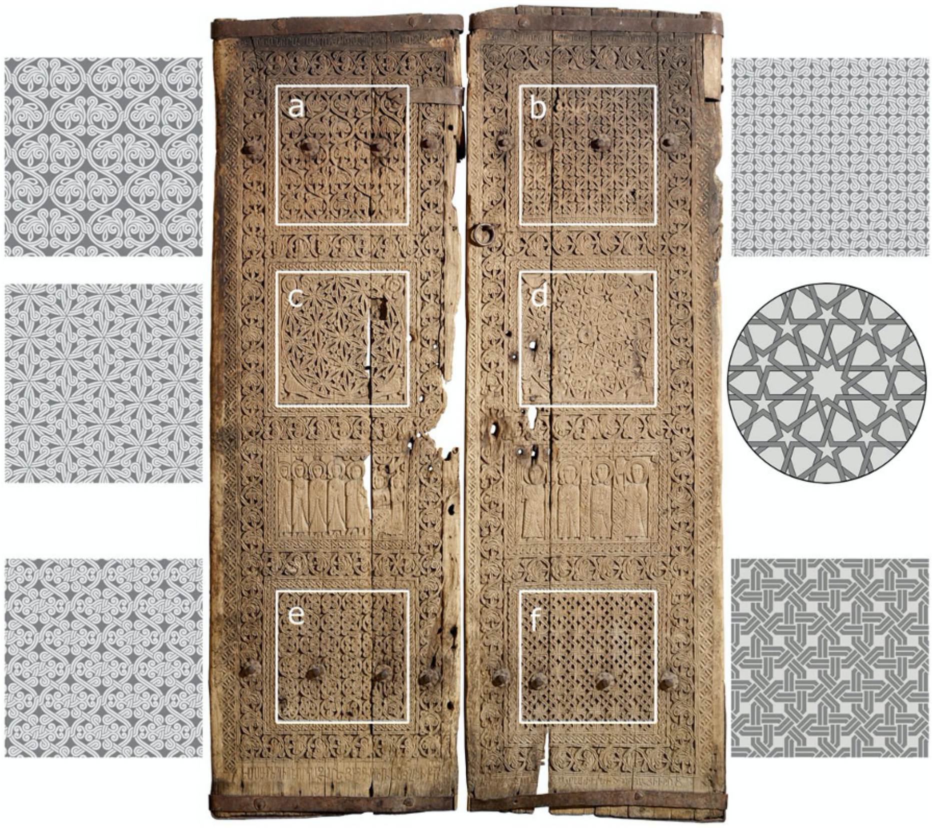

Figure 4. The wooden entrance door of the Surb Karapet Monastery near Mush, modern-day Turkey. Six areas of distinct symmetry are highlighted *a–f* with their abstractions shown on the left and right of the wooden door. The area *a* belongs to the group *c1m1*, *b* to *p4*, *c* to *p6*, the rosette *d* to *C10*, *e* to *c1m1* and *f* to *p4*. Symmetry analysis of the highlighted patterns is given later in text (photo courtesy Hrair Khatcherian).

Figure 3 shows a tombstone, a typical khachkar (*xačʻ* 'cross' and *kʻar* 'stone'). A common khachkar is a cross surrounded by elaborate patterns of leaves, grapes, pomegranates with biblical figures. This example with several supplementary crosses comes from the garden of the Aghjots Vank, the monastery of Saint Stephan of Gouht in the province of Ararat from the early 13th century. The upper part of the khachkar is covered with a pronounced ornament, while the left and right bands are also decorated. The upper part of the bands has a well-defined ornament with the same symmetry on each side. In the lower part of the bands, the width of each pattern is not large enough to define a periodic ornament. In the middle, however, there is a beautiful pattern with a distinct abstraction on the right. In the garden of the Armenian sacral buildings, there is a large collection of preserved khachkars (Kyurkchyan/Khatcherian 2010; Yevadian 2006).

Figure 4 illustrates the wooden main door of Surb Karapet (Saint John the Baptist) Monastery, one of the largest monasteries of Western Armenia, northwest of Mush, modern-day Turkey. The monastery was founded at the beginning of the 4th century and existed until 1915 when it was completely destroyed. On the figure we have outlined six areas *a–f*, which carry distinct ornamentation. The entire door area is covered with beautiful friezes.

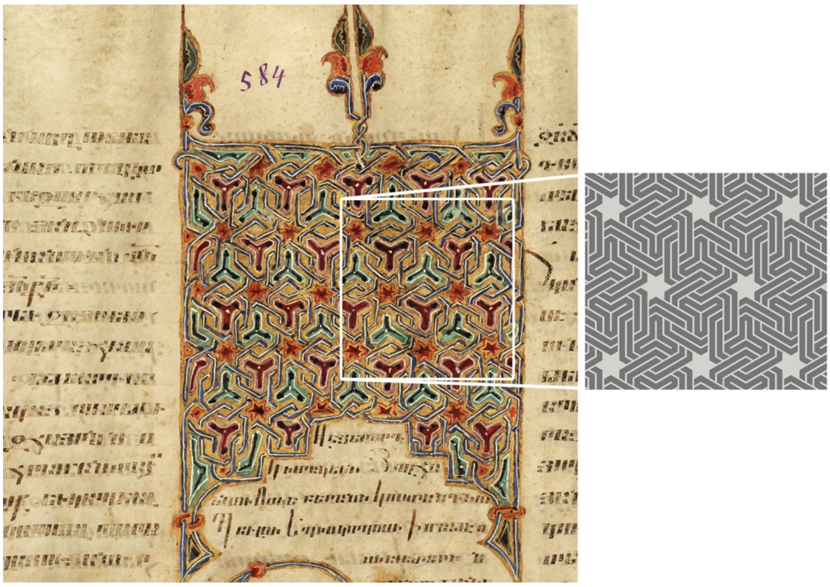

Figure 5. An Armenian miniature from the Matenadaran manuscript №7729, and the abstraction of its symmetry features typical for the group *p6*.

INDOOR TREASURES

So far, we have presented samples of some ornaments on outdoor objects. Rare examples can also be found as indoor treasures, such as on paper and fabric. Several Armenian examples are presented elsewhere (Kyurkchyan/Khatcherian 2010). Figure 5 shows a page from the Matenadaran manuscript №7729, Msho Tcharryntir (Homiliarium of Mush) 1200-1202. On the right there is an abstraction for easy identification of the symmetry.

Figure 6 is a printed fabric (see Kyurkchyan 2016). Observe the harmonious pattern with wavy lines in two perpendicular directions that mimic a circular motion. Armenian carpets are known worldwide, but they are not mentioned here at all. We do not know any mediaeval examples. Besides, like the *kilims*, they have a subsurface layer. Their layer structure cannot be considered within the wallpaper formalism, it must be classified as layered groups (Makovicky 2016).

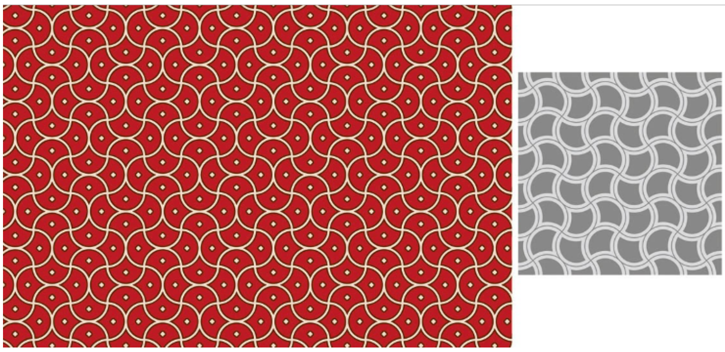

Figure 6. An Armenian block printed fabric (Kyurkchyan 2016) and its graphical abstraction

METHODS

The mathematical method for classifying periodic structures in two dimensions was originally used for the analysis of surface atomic structures, *viz.* surface crystallography (Fyodorov 1891; Schönflies 1891; Barlow 1894). Later, George Pólya (1924) suggested to apply it to patterns on fabric, wallpaper, and generally to any two-dimensional ornament. In this context, it was first applied by Müller (1944) to the decorations of the Alhambra Palace in Granada, Spain. We encounter rosettes as ornaments containing only the point-group symmetry. The frieze is a band with translation and *glide reflection* in one dimension, while the ornament covers the entire flat surface. The glide reflection *g* is a combination of a *mirror reflection m* over a line that follows a half translation along the same line, the so-called glide-reflection line. Since then, this method has been used primarily toanalyse Islamic ornaments and artefacts of several cultures.

Recently we sketched this classification scheme and applied it to the floor ornaments of the St. Marc's Basilica in Venice (Erbudak 2019). In short, the *motif* is the smallest part of the ornament, which facilitates the construction of the entire pattern, taking into account the symmetry group. It is also called *fundamental unit* or *generator*. To construct the *unit cell* of the pattern point-group symmetry operations the combination of mirror reflection and rotation about an axis, are applied to the motif. The unit cell is then embedded in one of the five appropriate *Bravais lattices* that periodically extends the unit cell into two-dimensional space using the *translation vectors* (Hammond 2015). The resulting 17 distinct planar patterns can be characterised with different nomenclatures (Schattschneider 1978); we follow the international one used by crystallographers and physicists (Hammond 2015). These patterns are two-dimensional ornaments commonly known as *wallpaper group*. In this scheme, the designation of each group follows a four-digit recipe recognised by the International Union of Crystallography: the first letter is a *p* or a *c*, i.e., *primitive* or *centred*, depending on the type of lattice. In the centred cell, there is an additional unit in the middle of the cell, otherwise not. The second digit is the order of rotation, i.e., the angular fraction of 2π by which the ornament must be rotated through an axis perpendicular to the plane of the ornament in order to map it to itself. It can be $n = 2, 3, 4$, and 6. $n = 1$ is the identity, i.e., a rotation of 360°. The third digit is a mirror reflection *m* or glide reflection *g* normal to the translation vectors, and the last digit is also an *m* or a *g* along the other major axis. If there is no mirror or glide, we write 1. Grouped into rotational symmetries, the following elements are distinguished: *p1*, *p1m1*, *p1g1*, *c1m1*, *p211*, *p2mm*, *p2mg*, *p2gg*, *c2mm*, *p4*, *p4mm*, *p4gm*, *p3*, *p3m1*, *p31m*, *p6*, *p6mm*. The groups *p1* and *p211* are embedded in an oblique lattice. Other rotations of order $n = 2$ require a rectangular lattice, $n = 4$ a square, and $n = 3$ and $n = 6$ hexagonal lattices. To analyse an unknown pattern, we find the unit cell that repeats itself to cover the entire surface with two-dimensional translations. The next step is to search for rotation properties of the pattern and reflection and glide lines. This procedure may require some experience but can easily be accomplished by visual inspection or computationally with a suitable computer code. Thus, one of the symmetry groups can be assigned to the ornament. The group-theoretical method has several advantages: It is systematic and objective, precisely defined, accurate and easy to document. The group-theoretical method is widely documented elsewhere (Schattschneider 1978; Hammond 2015). Previously, Washburn/ Crowe (1991) used the method to analyse cultural preferences of some indigenous tribes.

SYMMETRY PROPERTIES OF ARMENIAN ORNAMENTS

Kyurkchyan/Khatcherian (2010) contains a collection of aesthetic decorations and ornaments of Armenian mediaeval culture, collected over the last 30 years. For each pattern presented in the book, the exact origin, the year of creation and the material used are clearly indicated. We use these patterns in idealised form of graphics and identify the symmetry elements of two-dimensional ornaments, i.e., unit cells, translation vectors (blue arrows), mirror- (red lines) and glide-reflection (green lines) lines, and rotation axes (diamonds, triangles, squares, and hexagons for rotational orders 2, 3, 4, and 6, respectively).

In the following, examples of Armenian two-dimensional ornaments are presented. We find that ornaments with some symmetries are not available, and few others exist only as one or two examples. Figure 7 illustrates three patterns that are abstracted to reveal the inherent symmetry. These patterns consist of unit cells that repeat themselves in two dimensions. This repetition is the most basic symmetry in crystallography and is present in all seventeen groups. The repetition distance in two dimensions is given by the translation vector, which defines the *periodicity*. In the group *p1* the translation vectors do not necessarily have the same length and are not necessarily perpendicular to each other. Therefore, the unit cell generally has an *oblique* shape. Apart from the translations, there are no other symmetry properties in this group. In the figure, on the pattern to the left, blue arrows are the translation vectors that define the unit cell which is bounded by thin black lines. By chance, in the case it is a rectangle.

In the group *p1m1*, there is a mirror symmetry in one principal direction in addition to the translation in two directions. We could not find an example for this group in the Armenian ornamentation. Likewise, there is no example for the group *p1g1*. This group contains a glide reflection in one direction, as well as translations in two directions (for examples of these two groups, see Erbudak/Kyurkchyan 2019).

Figure 8 displays three ornaments in the group *c1m1*. The pattern on the left identifies symmetry properties, i.e., the unit cell spanned by the translation vectors (blue) and thin black lines. The lattice is a rhombus, which can be considered as part of the centred rectangle. The mirror- and glide-reflection axes (red and green lines, respectively) repeat themselves periodically parallel to each other and have the same distance to adjacent lines. This group contains no rotations, except for $n = 1$, i.e., 360°.

Figures 4a and 4e each contain a pattern of this symmetry. The overall impression of these ornaments of the group *c1m1* is the left-right gliding property along the vertical glide-reflection lines.

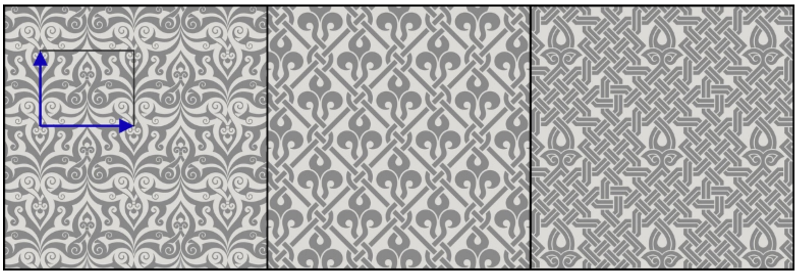

Figure 7. Three examples for patterns belonging to the group *p1*.

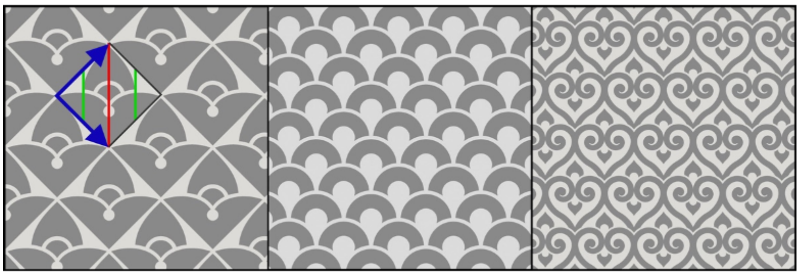

Figure 8. Three ornaments in the group *c1m1*

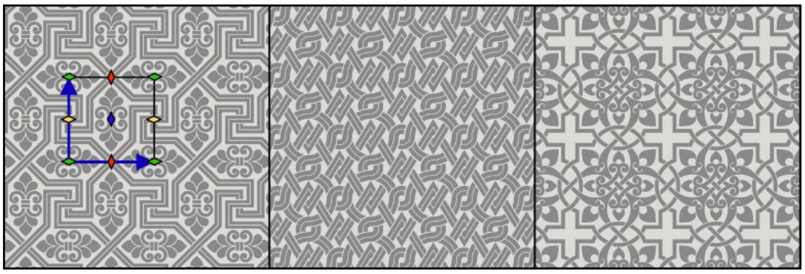

Figure 9. Three ornaments of the group *p21*

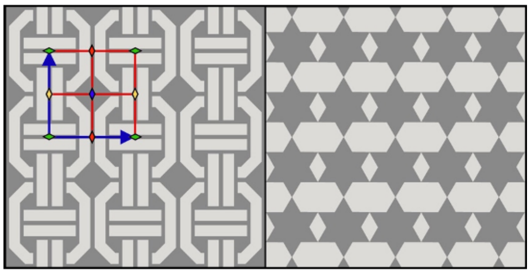

Figure 10. Two ornaments of the group *p2mm*

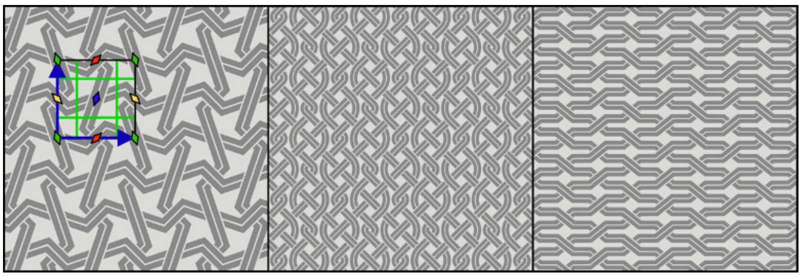

Figure 11. Three ornaments of the group *p2gg*. The symmetry operations are indicated on the left. Translation vectors and the thin black lines form the unit cell. Diamonds of different colours are the twofold-rotation axes, while green lines represent the glide-reflection lines.

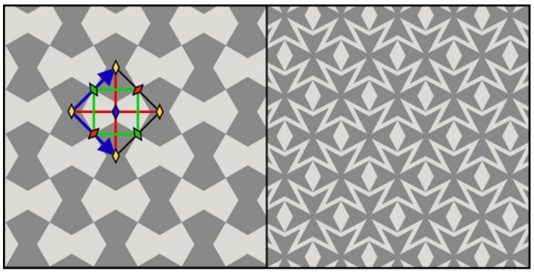

Figure 12. Two ornaments of the group *c2mm*. The symmetry operations are indicated on the left. Translation vectors and the thin black lines define the unit cell. Diamonds of different colours are the twofold-rotation axes, while red and green lines represent the mirror- and glide-reflection lines.

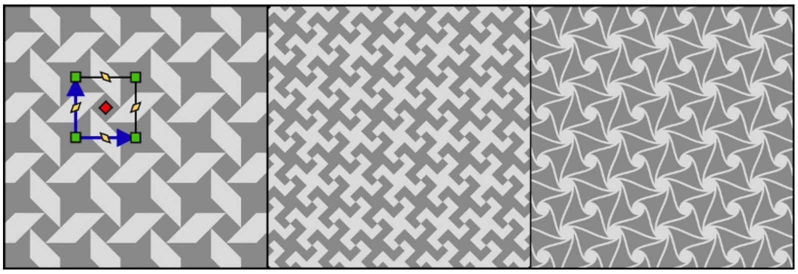

Figure 13. Three ornaments of the group *p4*. The symmetry operations are shown on the left. Translation vectors and the thin black lines define the unit cell. Diamonds represent the twofold-rotation axes, while small red and green squares stand for the fourfold-rotation axes.

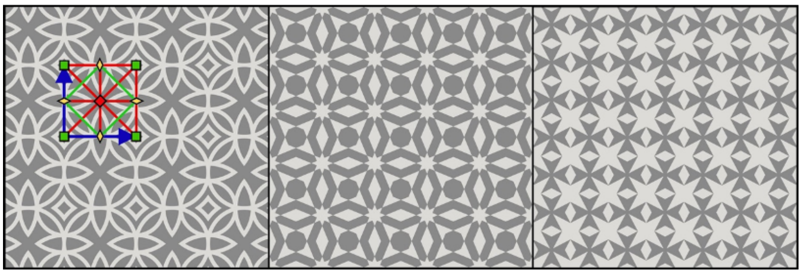

Figure 14. Three ornaments of the group *p4mm*. The symmetry operations are shown on the left. Translation vectors define the unit cell. Diamonds represent the twofold-rotation axes, while small red and green squares represent the fourfold-rotation axes.

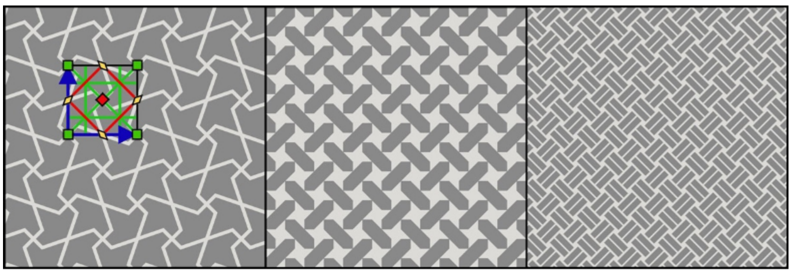

Figure 15. Three ornaments of the group *p4gm*. The symmetry operations are shown on the left. Translation vectors and the thin black lines define the unit cell. Yellow diamonds point to the twofold-rotation axes, while small red and green squares represent the fourfold-rotation axes.

Figure 9 shows three patterns of the group *p211*. The translation vectors (blue arrows in the left pattern) are perpendicular to each other, but do not necessarily have the same length, i.e., the lattice is rectangular. $n = 2$ implies twofold symmetry, and there are four different types of twofold-symmetry axes marked with diamonds of different colours. The yellow and red rotation centres are halfway between the corners of the unit cell. Note that you can place the translation vectors with the unit cell anywhere on the surface of the pattern, while the centres of rotation are set. There are no mirrors or glide reflections.

Figure 10 presents two ornaments in the group *p2mm*. The symmetry properties of the group are superimposed on the pattern on the left. The translation vectors are perpendicular to each other, the lattice type is primitive rectangular. There are four different types of twofold-symmetry axes marked with diamonds of different colours. We identify two types of mirror reflections, horizontal and vertical. All centres of rotation are located on the mirror-reflection lines. The pattern on the left contains several bands in two directions. This graphic technique gives the ornament a distinct three-dimensional appearance and a mysterious depth. It is a pseudodynamic effect widely used by other cultures and well demonstrated for Islamic ornaments (Bonner 2017).

In the collection of Armenian ornaments (see Kyurkchyan/Khatcherian 2010), we could not find a pattern in the group *p2mg*. But there are several ornaments that belong to the group *p2gg*. Three of them are presented in Figure 11. The symmetry elements of the group are superimposed on the pattern on the left. The translation vectors (blue) define the unit cell. They are perpendicular to each other, which makes the unit cell rectangular, as well as the lattice type. We distinguish four different types of twofold-rotation axes, all marked with diamonds of different colours. There are two sets of glide reflection lines that run horizontally and vertically through the midpoints of the centres of rotation. All three patterns are formed by discontinuous bands. We have seen this construction principle on the marble stones of the St. Marc's Basilica (Lazzarini 2012). Although extremely elaborate and difficult to realise, this technique gives the ornament a pseudo three-dimensional appearance.

Figure 12 shows two patterns of the group *c2mm*, while the symmetry properties of the group are shown in the left pattern. A centred pattern is a rhombus, as we can see here, which is actually a part of a rectangle. The unit cell is the smallest entity that repeats itself over the entire surface. Therefore, the representation of the entire centred rectangle, whose perpendicular sides are as long as the red mirror-reflection lines, is omitted. We observe that there are four different twofold-rotation axes in the unit cell, each indicated as a diamond of different colours. A pattern in the group *c2mm* has two perpendicularly aligned mirror (red) and glide (green) reflections. They run periodically in alternation with each other.

Figure 13 depicts three ornaments belonging to the group *p4*. The translation vectors (blue arrows) have the same length and are perpendicular to each other, i.e., the lattice is square. This pattern has two different types of rotations of order 4, i.e., 90° rotations marked with green and red squares. One type is at the corners, the other in the middle of the square unit cell. The axes for twofold rotations are halfway between the centres for fourfold rotation. This group *p4* contains translations and rotations, but no mirror or glide reflections. Almost half of the available ornaments belong to this group.

The highlighted areas of Figure 13 in the center and below, each display examples of the group *p4*. We also show the abstraction of individual patterns on the right. Both are difficult to carve in this way to convey the three-dimensional impression through discontinuous bands. Similarly, the Figures 4b and 4f each show beautiful wood carvings with a pattern in the group *p4*.

Figure 14 illustrates three ornaments belonging to the group *p4mm*. The translation vectors have the same length and are perpendicular to each other, i.e., the lattice is square. This group has two different types of rotations of order 4. The axes of twofold symmetry are halfway between the centres for fourfold rotation. There are horizontal and vertical as well as 45°-inclined mirror reflections, so that four reflection axes run through the centres of fourfold symmetry. Parallel to the mirror-reflection axes, which are inclined by 45°, we identify two pairs of glide reflections in two directions perpendicular to each other. They meet at the twofold-rotation axes. These properties define the symmetry group *p4mm*. The ornament on the cape of the Saint Hamazasp shown in Figure 2 is a typical representative of this group.

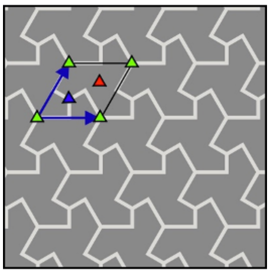

Figure 16. An ornament of the *p3* group. The symmetry operations are superimposed on the pattern. Translation vectors and the thin black lines define the unit cell. Three different threefold axes are represented as coloured triangles.

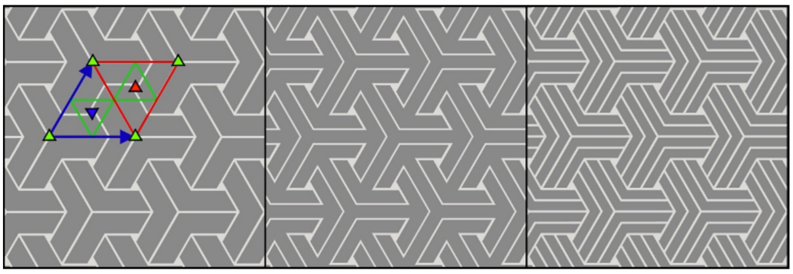

Figure 17. Three ornaments of the *p31m* group. The symmetry operations are superimposed on the pattern on the left. Translation vectors define the unit cell. Three different threefold axes are represented as green triangles. Red and green lines stand for the mirror-and glide-reflection lines.

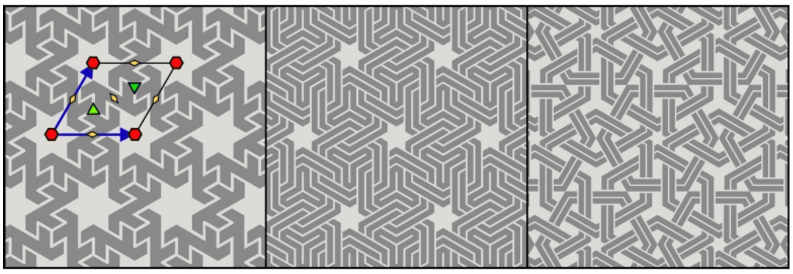

Figure 18. Three ornaments of the group *p6*. The symmetry operations are superimposed on the pattern on the left. Translation vectors and thin black lines define the unit cell. Red hexagons are the sixfold-rotation axes. Threefold-rotation axes are represented as green triangles. Yellow diamonds indicate the twofold-rotation axes.

Three patterns in Figure 15 are ornaments belonging to the group *p4gm*. The translation vectors have the same length and are perpendicular to each other, i.e., the lattice is square. This group has two different centres of fourfold symmetry. The mirror reflections take place in two perpendicular directions, which run through twofold-rotation centres and not through the fourfold-symmetry axes. Hence, they run at 45° to the translation vectors. There are four glide-reflection lines in two pairs perpendicular to each other. One pair is parallel to the translation vectors and the other rotated by 45°. The first pair runs halfway between twofold- and fourfold-rotation axes. The other pair of glide reflection lines is parallel to the mirror-reflection lines and passes through fourfold-symmetry centres. The decoration on the printed fabric shown in Figure 6 is a nice example of the group *p4gm*.

The simplest member of the group $n = 3$ is a pattern with symmetry *p3*, illustrated in Figure 16. There is only one single pattern in our (AK) collection that belongs to the *p3* group. The translation vectors have the same length and are separated by 60°; the unit cell is a rhombus and the lattice is hexagonal. The rotation of $n = 3$ is manifested by three different types of threefold-rotation axes, blue, red, and green.

There are two more representative groups with the rotational symmetry $n = 3$: the *p3m1* and *p31m*. They are closely related to each other: In *p3m1* all threefold-symmetry axes lie on mirror-reflection lines, while in *p31m* it is not necessarily so. No member of the *p3m1* group could be found in the arsenal of Armenian ornaments (Kyurkchyan/Khatcherian 2010).

In Figure 17, we show ornaments with translation vectors of equal length and an opening angle of 60°. The lattice is hexagonal. The mirror lines are parallel to the translation vectors. As can be seen in the figure, this group contains three different types of threefold-rotation axes. There is at least one rotation axis which is not on a mirror-reflection line. These two properties are the main characteristics of the group *p31m*. There are also glide-reflection axes at intervals of 60° that run through the midway between threefold-symmetry axes. The ornaments shown here give a fascinating three-dimensional impression.

Figure 18 presents three ornaments of the group *p6* with intriguing details. This group includes 60° rotations ($n = 6$, red hexagons), two threefold- (green triangles) and three twofold-rotation (yellow diamonds) axes. The threefold-symmetry axes are located at midpoints of three sixfold-symmetry centres forming an equilateral triangle. The twofold-symmetry axes are located at midpoints of each two sixfold-symmetry axes. All threefold rotations are related to each other by rotations of 60°, as are the twofold rotations. There are no mirror or glide reflections. The lattice group is hexagonal.

The upper ornament in Figure 3 is a simple example of the group *p6* with ribbons that give the pattern a three-dimensional look like wool knitting. Likewise, in Figure 4c there is a pattern of the group *p6* in a limited area enclosed by a circle as well as the miniature shown in Figure 5.

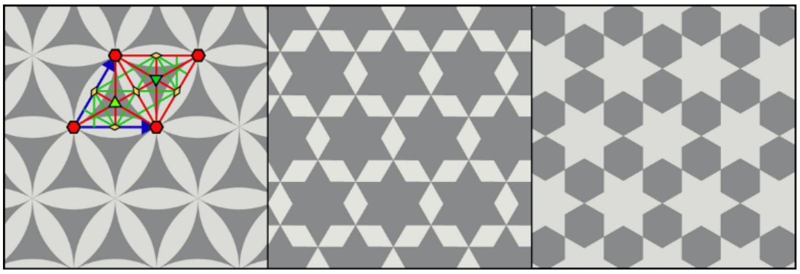

Figure 19. Three ornaments of the *p6mm* group. The symmetry operations are superimposed on the pattern on the left. Translation vectors define the unit cell. Red hexagons are the sixfold-rotation axes. Threefold-rotation axes are indicated as green triangles. Yellow diamonds represent the twofold-rotation axes. Red and green lines stand for the mirror- and glide-reflection lines.

Figure 19 finally shows three sixfold-symmetric ornaments of the group *p6mm*. Despite its simplicity, this group contains, besides the 60° rotations (red hexagons), two different types of threefold- (green triangles) and three different types of twofold-rotation (yellow diamonds) axes. There are mirror-reflection lines running at 60° intervals through the sixfold-symmetry centres. The threefold-symmetry axes are located at midpoints of each three sixfold-symmetry centres, forming an equilateral triangle. The twofold-symmetry axes are located at midpoints of each two sixfold-symmetry centres. There are six glide reflections at every 60°, which run through twofold-symmetry axes. We consider that the mirror reflections exclude overlaps with glide reflections. The lattice group is hexagonal; the angle between the translation vectors (blue arrows) is 60°.

Figure 4d contains a beautiful rosette with tenfold-rotation symmetry C_{10}, which can also be found in other cultures (Bonner 2017). The inspection of the pattern shows how the area is perfectly covered aperiodically with corresponding figures, including fivefold stars. Although perfect in point-group symmetry, this pattern is not periodic and cannot be treated within the crystallographic plane groups.

SUMMARY

We have analysed over 120 ornaments consisting of graphical artwork found on Armenian sacral buildings, books, and fabric (Bachmann 1913; Kyurkchyan/Khatcherian 2010; Kyurkchyan 2016). We have classified the objects into 17 wallpaper groups. Our study shows that, ordered by rotational symmetry, the occurrence of ornaments with $n = 1$ is about 8 %, with $n = 2$ is about 14 %, with $n = 3$ is about 3 %, with $n = 4$ is about 66 %, and with $n = 6$ is about 9 %. The results are summarised in Table 1. We consider this distribution to be balanced, i.e., each rotation group is represented with a dominance of fourfold symmetry. There can be several reasons for the rich occurrence of the group *p4*. Practically, square building blocks on which the ornaments are carved are the most efficient to be integrated into the structure of a building, as can be seen in Figure 1.

Symmetry Group	Occurrence	%
p1	7	5.7
p1m1	0	0
p1g1	0	0
c1m1	3	2.4
p211	5	4.1
p2mm	2	1.6
p2mg	0	0
p2gg	8	6.5
c2mm	2	1.6
p3	1	0.8
p3m1	0	0
p31m	3	2.4
p4	61	49.6
p4mm	16	13.0
p4gm	4	3.3
p6	6	4.9
p6mm	5	4.1

Table 1. Occurrence of each symmetry group in an ensemble of 123 ornaments. Rotational orders are combined. Each event is specified as a number and percentage of the total number of available ornaments.

CONCLUSIONS

Our example of a religious building, the Surb Khach Church, has human and animal figures on the exterior walls, which are furnished with accessories rich in rosettes, friezes, and ornaments. Also carved in stone are the khachkars, the gravestones with similarly rich ornaments. Wooden parts of mediaeval buildings, such as doors also bear typical decorations. In addition to these objects of communal life, also those for individuals, books, or fabrics, similarly contain patterns.

We have observed that many patterns have discontinuous bands that enrich the ornament with a three-dimensional spatiality. This quality is a testimony to a high-level craftsmanship, namely the ability to imagine and create spatial objects. This virtue in stone masonry is the most striking feature of the Armenian craftsmen. The extension of a pattern into the imaginary three-dimensional space is achieved by carving stones in Armenia, while in Byzantium similar effects are realised by inlaying marble (Lazzarini 2012) and glazed ceramic tiles in the Islamic world (Bonner 2017).

Although the fourfold symmetry dominates, the distribution of the different symmetries is otherwise balanced. In contrast, the floor ornaments of St. Marc's Basilica have neither a single threefold-symmetrical pattern among the more than 750 samples observed nor a single sixfold symmetric *p6mm* (Erbudak 2019). The balanced symmetry distribution of the Armenian ornaments can be attributed to the interaction of Armenia with the Arab world in the south, Byzantine culture in the west, and with the Seljuks and Persian culture in the east. To the best of our knowledge, no systematic mathematical symmetry analysis of the Seljuk or Persian art patterns has been carried out to date, with the exception of an excellent geometrical analysis (Schneider 1980). The material is sparse; some reports mention mainly wall works of art with tetragonal symmetry (Makovicky 2016; Bonner 2017). The Islamic work of art, on the other hand, has many threefold- and sixfold-symmetrical patterns (Bourgoin 1973). Armenia's geographical location along the Silk Road has inspired Armenian art from all directions. This observation provides an indication of the cultural interaction of peoples and supports the relevance of the mathematical analysis of ornaments in pursuit of such interactions.

Several objects mentioned in this work are no longer in their place, like the door of the Surb Karapet Monastery shown in Figure 4. All the artworks listed in Kyurkchyan/Khatcherian (2010) are located either in Armenia or in Turkey (Ani, Akhtamar, and Van). They are outdoors and exposed to all kinds of natural hazards. They can disappear over the years.

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